EE105 – Fall 2014
Microelectronic Devices and Circuits

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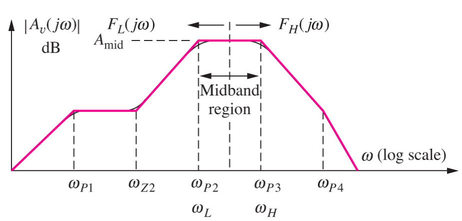
Amplifier Frequency Response
Transfer Function Analysis



$$F_L(s)=\frac{(s+\omega^L_{Z1})(s+\omega^L_{Z2})\cdots(s+\omega^L_{Zk})}{(s+\omega^L_{P1})(s+\omega^L_{P2})\cdots(s+\omega^L_{Pk})}$$

$$F_H(s)=\frac{\left(1+\frac{s}{\omega^H_{Z1}}\right)\left(1+\frac{s}{\omega^H_{Z2}}\right)\cdots\left(1+\frac{s}{\omega^H_{Zl}}\right)}{\left(1+\frac{s}{\omega^H_{P1}}\right)\left(1+\frac{s}{\omega^H_{P2}}\right)\cdots\left(1+\frac{s}{\omega^H_{Pl}}\right)}$$

$$A_v(s)=\frac{N(s)}{D(s)}=\frac{a_0+a_1s+a_2s^2+...+a_ms^m}{b_0+b_1s+b_2s^2+...+b_ns^n}$$

$$A_v(s)=A_{mid}F_L(s)F_H(s)$$

A_{mid} is the midband gain between the lower and upper cutoff frequencies ω_L and ω_H.

$$|F_H(j\omega)|\to 1 \text{ for } \omega<<\omega^H_{Zi},\omega^H_{Pi},\ \text{j}=1...l$$

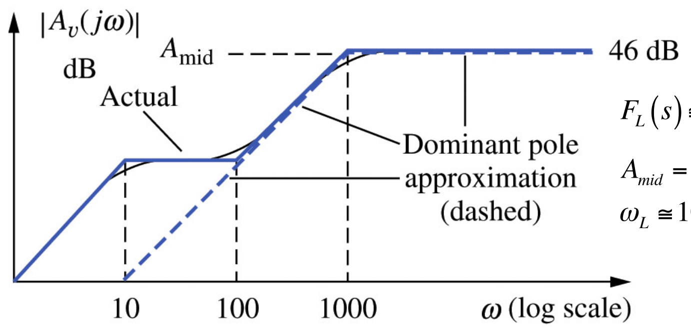
$$\therefore A_L(s)\cong A_{mid}F_L(s)$$

$$|F_L(j\omega)|\to 1 \text{ for } \omega>>\omega^L_{Zj},\omega^L_{Pj},\ \text{j}=1...\text{k}$$

$$\therefore A_H(s)\cong A_{mid}F_H(s)$$

Low Frequency Dominant Pole Approximation

- A ***Dominant Pole*** **exists if one of the low frequency poles is much larger than the others.**
  - **In the graph below case ω = 1000 rad/sec is a dominant pole. All other poles and zeros are at low enough frequencies that they do not affect the lower cutoff frequency ω_L.**



$$F_L(s) \cong \frac{s}{s+1000}$$

$$A_{mid} = 200$$

$$\omega_L \cong 1000\ rad/\sec$$

Lower Cutoff Frequency Calculations

If there is no dominant pole at low frequencies, the poles and zeros interact to determine the lower cutoff frequency ω_L.
For example, suppose:

$$A_L(s) = A_{mid}F_L(s) = A_{mid}\frac{(s+\omega_{Z1})(s+\omega_{Z2})}{(s+\omega_{P1})(s+\omega_{P2})}$$

For $s = j\omega_L$, $\left|A_L(j\omega_L)\right| = \frac{A_{mid}}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{(\omega_L^2+\omega_{Z1}^2)(\omega_L^2+\omega_{Z2}^2)}{(\omega_L^2+\omega_{P1}^2)(\omega_L^2+\omega_{P2}^2)}}$$

$$\frac{1}{2} = \frac{\omega_L^4+(\omega_{Z1}^2+\omega_{Z2}^2)\omega_L^2+\omega_{Z1}^2\omega_{Z2}^2}{\omega_L^4+(\omega_{P1}^2+\omega_{P2}^2)\omega_L^2+\omega_{P1}^2\omega_{P2}^2}$$

Lower cutoff frequency ω_L will be greater than all the individual pole zero frequencies.

$$\therefore \omega_L \cong \sqrt{\omega_{P1}^2+\omega_{P2}^2-2\omega_{Z1}^2-2\omega_{Z2}^2}$$

In general, for n poles and n zeros,

$$\omega_L \cong \sqrt{\sum_n \omega_{Pn}^2 - 2\sum_n \omega_{Z1}^2}$$

Dominant Pole Example

- **Problem: Find midband gain, $F_L(s)$ and f_L for**

$$A_L(s) = 2000\frac{s\left(\frac{s}{100}+1\right)}{(0.1s+1)(s+1000)}$$

- **Analysis: Rearranging the given transfer function into standard form,**

$$A_L(s) = 200\frac{s(s+100)}{(s+10)(s+1000)} = A_{mid}F_L(s) \rightarrow A_{mid} = 200 \qquad F_L(s) = \frac{s(s+100)}{(s+10)(s+1000)}$$

Zeros: s = 0 and s = -100   *Poles*: s = -100 and s = -1000

The poles and zeros are all widely separated. $\therefore A_L(s) \cong 200\frac{s}{s+1000}$

The dominant pole is at $\omega = 1000$ and $f_L \cong \frac{1000}{2\pi} = 159\ Hz.$

The more exact calculation is $f_L = \frac{1}{2\pi}\sqrt{10^2 + 1000^2 - 2\times 0^2 - 2\times 100^2} = 158\ Hz$

High-Frequency Dominant Pole

- **The lowest of all high frequency poles is called the dominant high-frequency pole.**

$$A_H(s) \cong A_{mid}F_H(s)$$

$$F_H(s) \cong \frac{1}{1+(s/\omega_{P3})} \qquad \omega_H \cong \omega_{P3}$$

- **If there is no dominant pole at high frequencies, the poles and zeros interact to determine ω_H.**

$$A_H(s) = A_{mid}F_H(s) = A_{mid}\frac{\left(1+\frac{s}{\omega_{Z1}}\right)\left(1+\frac{s}{\omega_{Z2}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)}$$

For $s = j\omega_H$, $\left|A_{mid}(j\omega_H)\right| = \frac{A_{mid}}{\sqrt{2}}$

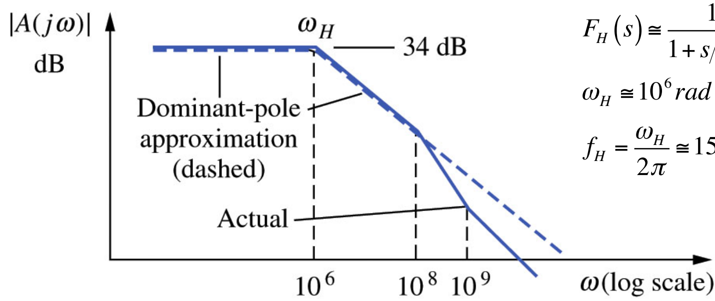
$$\frac{1}{\sqrt{2}} = \sqrt{\frac{\left[1+\left(\frac{\omega_H}{\omega_{Z1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{Z2}}\right)^2\right]}{\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]}}$$

$$\frac{1}{\omega_H} \cong \sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} - \frac{2}{\omega_{Z1}^2} - \frac{2}{\omega_{Z2}^2}}$$

For the general case of n poles and n zeros,

$$\frac{1}{\omega_H} \cong \sqrt{\sum_n \frac{1}{\omega_{Pn}^2} - 2\sum_n \frac{1}{\omega_{Zn}^2}}$$

High Frequency Dominant Pole Approximation



$$A_H(s) = A_{mid} F_H(s)$$

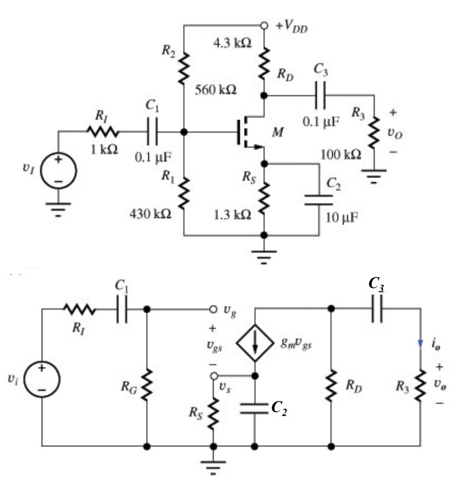
$$A_{mid} = 200$$

$$F_H(s) \cong \frac{1}{1 + s/10^6} = \frac{10^6}{s + 10^6}$$

$$\omega_H \cong 10^6\, rad/\sec$$

$$f_H = \frac{\omega_H}{2\pi} \cong 159\ kHz$$

Low-Frequency Poles and Zeros Direct Calculation: C-S Amplifier



$$V_o(s) = I_o(s) R_3 = -g_m V_{gs}(s) \frac{R_D}{R_D + (1/sC_3) + R_3} R_3$$

$$V_o(s) = -g_m (R_D \| R_3) \frac{s}{s + \frac{1}{C_3(R_D + R_3)}} V_{gs}(s)$$

$$V_g(s) = \frac{sC_1 R_G}{sC_1(R_I + R_G) + 1} V_i(s)$$

Node Eq. at Source: $g_m(V_g - V_s) = \frac{V_s}{R_s} + sC_2 V_s$

$$V_{gs}(s) = V_g - V_s = \frac{s + \frac{1}{R_S C_2}}{s + \frac{1}{\left(\frac{1}{g_m} \| R_S\right) C_2}} V_g(s)$$

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = A_{mid} F_L(s)$$

$$A_{mid} = -g_m (R_D \| R_3) \frac{R_G}{R_I + R_G}$$

Low-Frequency Poles and Zeros Direct Calculation: C-S Amplifier (cont.)

$$F_L(s) = \frac{s^2\left(s + \frac{1}{R_S C_2}\right)}{\left[s + \frac{1}{(R_I + R_G)C_1}\right]\left[s + \frac{1}{(1/g_m \| R_S)C_2}\right]\left[s + \frac{1}{(R_D + R_3)C_3}\right]}$$

The three zero locations are: $s = 0,\ 0,\ -1/R_S C_2$

The three pole locations are: $s = -\frac{1}{(R_I + R_G)C_1},\ -\frac{1}{(1/g_m \| R_S)C_2},\ -\frac{1}{(R_D + R_3)C_3}$

Each independent capacitor in the circuit contributes one pole and one zero. Series capacitors C_1 and C_3 contribute the two zeros at s = 0 (dc), blocking propagation of dc signals through the amplifier. The third zero due to the parallel combination of C_2 and R_S occurs at frequency where signal current propagation through the MOSFET is blocked (output voltage is zero).